internal and external energy sources relative to the body considered; W, specific power supplied to a unit volume of the body from external and internal energy sources; U, specific internal energy of the body; V, volume of the body; v, carrier propagation velocity; S, number of species of body particles; λ and a, thermal conductivity and thermal diffusivity; c, specific heat; ρ, body density; F, total effective cross section of particle absorption of unit volume; ε, energy emission coefficient; $\varepsilon_\nu$, emission coefficient of photons of frequency ν by body particles; $n_{i\nu}$, density of particles found at frequency ν at the i-th energy level; h, Planck's constant; Fo, Fourier number; and Ei, integral exponential function.

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# TEMPERATURE FIELD IN A HALF-SPACE WITH A FOREIGN INCLUSION


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A stationary temperature field is studied in a half-space containing a heat-liberating disclike foreign inclusion of small size. Convective heat transfer with the external medium is realized through its boundary surface.


Let us consider an isotropic half-space containing a foreign cylindrical inclusion of radius R and height $l$ at a distance d from its boundary surface, where uniformly distributed internal heat sources of intensity $q_0$ act. Let the body under consideration be referred to a cylindrical coordinate system. We place the origin at the center of the inclusion. Convective heat transfer with the external medium of temperature $t_c$ is given at the boundary surface $z = \ell - d$.

To determine the stationary temperature field, we have the heat-conduction equation [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(r, z)\frac{\partial\Theta}{\partial r}\right]+\frac{\partial}{\partial z}\left[\lambda(r, z)\frac{\partial\Theta}{\partial z}\right]=-q_0S_-(R-r)N(z), \tag{1}$$

where $\Theta = t - t_c$; $N(z) = S_-(z+l) - S_+(z-l)$.

The boundary conditions are written in the form

$$\lambda_1\frac{\partial\Theta}{\partial z}=\alpha_z\Theta \text{ for } z=-l-d,\ \Theta=0 \text{ for } r,\ z\to\infty,$$
$$\frac{\partial\Theta}{\partial r}=0 \text{ for } r\to\infty. \tag{2}$$


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We represent the heat-conduction coefficient in the form

$$\lambda(r, z) = \lambda_1 + (\lambda_0 - \lambda_1) S_-(R-r) N(z). \tag{3}$$

Substituting (3) into (1) and differentiating according to the rule set up in [2], we arrive at the equation

$$\Delta\Theta = (K_\lambda - 1)\left\{\frac{\partial\Theta}{\partial r}\Big|_{r=R-0}\delta_+(r-R)N(z) - \left[\frac{\partial\Theta}{\partial z}\Big|_{z=-l+0}\delta_-(z+l) - \frac{\partial\Theta}{\partial z}\Big|_{z=l-0}\delta_+(z-l)\right]S_-(R-r)\right\} - \frac{q_0}{\lambda_0}S_-(R-r)N(z), \tag{4}$$

where

$$\delta_\pm(x) = \frac{dS_\pm(x)}{dx}.$$

The exact solution of the differential equation (4) can be obtained by the method proposed in [1] by using the representation of the temperature on the inclusion boundaries in the form of Fourier series. However, for small size inclusions $\left(\frac{l}{d} = \frac{R}{d} \leqslant \frac{1}{20}\right)$ the problem can be simplified considerably by assuming that the excess temperature over the inclusion thickness varies according to a linear law

$$\Theta(r, z) = \vartheta_z + \frac{z}{l}\vartheta_z^* \tag{5}$$

and equals its integral characteristic [3] along the radius

$$\vartheta_r = \frac{2}{R^2}\int_0^R r\Theta dr. \tag{6}$$

Here

$$\vartheta_z = \frac{1}{2l}\int_{-l}^{l}\Theta dz;\quad \vartheta_z^* = \frac{3}{2l^2}\int_{-l}^{l} z\Theta dz. \tag{7}$$

Then using (5)-(7), we obtain the following differential equation instead of (4)

$$\Delta\Theta = (K_\lambda - 1)\left\{\left[\frac{d\vartheta_z}{dr} + \frac{z}{l}\frac{d\vartheta_z^*}{dr}\right]\Big|_{r=R-0}\delta_+(r-R)N(z) - \left[\frac{d\vartheta_r}{dz}\Big|_{z=-l+0}\delta_-(z+l) - \frac{d\vartheta_r}{dz}\Big|_{z=l-0}\delta_+(z-l)\right]S_-(R-r)\right\} - \frac{q_0}{\lambda_0}S_-(R-r)N(z). \tag{8}$$

Applying the Hankel integral transform in the coordinate r to (8) and the boundary conditions (2), we arrive at an ordinary differential equation with constant coefficients

$$\frac{d^2\bar\Theta}{dz^2} - \xi^2\bar\Theta = R\left\{(K_\lambda - 1)\left[\left(\frac{d\vartheta_z}{dr} + \frac{z}{l}\frac{d\vartheta_z^*}{dr}\right)\Big|_{r=R-0}J_0(R\xi)N(z) - \left[\frac{d\vartheta_r}{dz}\Big|_{z=-l+0}\delta_-(z+l) - \frac{d\vartheta_r}{dz}\Big|_{z=l-0}\delta_+(z-l) + \frac{q_0}{\lambda_0}N(z)\right]\frac{J_1(R\xi)}{\xi}\right\} \tag{9}$$

and the following boundary conditions

$$\lambda_1 \frac{d\bar{\Theta}}{dz} = \alpha_z \bar{\Theta} \quad \text{for } z = -l-d, \ \bar{\Theta} = 0 \text{ for } \quad z \to \infty, \tag{10}$$

where

$$\bar{\Theta} = \int_0^\infty r\Theta J_0(r\xi)\, dr.$$

Solving the boundary-value problem (9) and (10) and then going over to originals by means of the inversion formula, we obtain the expression

$$T(r, z) = (1-K_\lambda)\left[\frac{d\vartheta_z}{dr}\bigg|_{r=R-0} F_1(r, z) + \frac{d\vartheta_z^*}{dr}\bigg|_{r=R-0} F_2(r, z) + \right.$$
$$\left. + \frac{d\vartheta_r}{dz}\bigg|_{z=l-0} F_3(r, z) - \frac{d\vartheta_r}{dz}\bigg|_{z=-l+0} F_4(r, z)\right] + \frac{1}{R} F_5(r, z), \tag{11}$$

where

$$T(r, z) = \frac{\lambda_0}{R^2 q_0}\Theta(r, z);$$

$$F_i(r, z) = \int_0^\infty J_0(R\xi) J_0(r\xi) f_i(z, \xi)\, d\xi, \ i = 1, 2;$$

$$F_j(r, z) = \int_0^\infty J_1(R\xi) J_0(r\xi) f_j(z, \xi)\, d\xi, \ j = 3, 4, 5;$$

$$f_1(z, \xi) = \frac{1}{\xi}[\operatorname{sh} l\xi \varphi_1(z, \xi) + \varphi_2(z, \xi)];$$

$$f_2(z, \xi) = \frac{1}{\xi}\left[\left(\frac{\operatorname{sh} l\xi}{l\xi} - \operatorname{ch} l\xi\right)\varphi_1(z, \xi) + \varphi_3(z, \xi)\right];$$

$$f_3(z, \xi) = \frac{1}{2\xi}[\exp(-l\xi)\varphi_1(z, \xi) - \varphi_4(z, \xi)];$$

$$f_4(z, \xi) = \frac{1}{2\xi}[\exp l\xi \varphi_1(z, \xi) - \varphi_5(z, \xi)]; \ f_5(z, \xi) = \frac{1}{\xi} f_1(z, \xi);$$

$$\varphi_1(z, \xi) = \exp \xi z + \alpha(\xi)\exp[-\xi(2(l+d)+z)]; \ \varphi_2(z, \xi) = N(z) -$$
$$- \operatorname{ch}\xi(z+l) S_-(z+l) + \operatorname{ch}\xi(z-l) S_+(z-l);$$

$$\varphi_3(z, \xi) = \frac{z}{l} N(z) + \left[\operatorname{ch}\xi(z+l) - \frac{\operatorname{sh}\xi(z+l)}{l\xi}\right] S_-(z+l) +$$
$$+ \left[\operatorname{ch}\xi(z-l) + \frac{\operatorname{sh}\xi(z-l)}{l\xi}\right] S_+(z-l);$$

$$\varphi_4(z, \xi) = 2\operatorname{sh}\xi(z-l) S_+(z-l); \ \varphi_5(z, \xi) = 2\operatorname{sh}\xi(z+l) S_-(z+l);$$

$$\alpha(\xi) = \frac{\lambda_1\xi - \alpha_z}{\lambda_1\xi + \alpha_z}; \ \frac{d\vartheta_z}{dr}\bigg|_{r=R-0} = \frac{\Delta_1}{\Delta_5}; \ \frac{d\vartheta_z^*}{dr}\bigg|_{r=R-0} = \frac{\Delta_2}{\Delta_5};$$

$$\frac{d\vartheta_r}{dz}\bigg|_{z=l-0} = \frac{\Delta_3}{\Delta_5}; \ \frac{d\vartheta_r}{dz}\bigg|_{z=-l+0} = \frac{\Delta_4}{\Delta_5}; \ \Delta_5 = \det[A_{kn}];$$

$$A_{kn} = \int_0^\infty J_0(R\xi) J_1(R\xi)\psi_{kn}(\xi)\, d\xi, \ A_{km} = \int_0^\infty J_1^2(R\xi)\psi_{km}(\xi)\, d\xi, \ A_{nn} = 1 +$$
$$+ \int_0^\infty J_0(R\xi) J_1(R\xi)\psi_{nn}(\xi)\, d\xi, \quad A_{mm} = 1 + \int_0^\infty J_1^2(R\xi)\psi_{mm}(\xi)\, d\xi,$$

$$k=1,\ 2,\ 3,\ 4,\ n=1,\ 2,\ m=3,\ 4,\ k\neq n,\ m;\ \psi_{11}(\xi)=2K\psi_{11}^{*}(\xi);$$

$$\psi_{12}(\xi)=\frac{K}{\xi}\alpha(\xi)\beta(-2d\xi)\left[\beta_1^-(-4l\xi)+\frac{1}{l\xi}[(2\beta(-2l\xi)-1-\beta(-4l\xi))\right];$$

$$\psi_{13}(\xi)=-\frac{K}{\xi}\beta_1^-(-2l\xi)\beta_2^+[\alpha(\xi),\ -2(l+d)\xi];$$

$$\psi_{14}(\xi)=\frac{K}{\xi}\beta_1^-(-2l\xi)\beta_2^+[\alpha(\xi),\ -2d\xi];\ \ \psi_{21}(\xi)=3K\psi_{21}^{*}(\xi);$$

$$\psi_{22}(\xi)=\frac{6K}{\xi}\left\{1-\frac{2}{3}l\xi+\frac{1}{l}\left[\left(l+\frac{1}{\xi}\left(2+\frac{1}{l\xi}\right)\right)\beta(-2l\xi)-\frac{1}{l\xi^2}\right]-\right.$$

$$-\alpha(\xi)\beta(-2d\xi)\left[0{,}5\beta_1^+(-4l\xi)+\beta(-2l\xi)+\frac{1}{l\xi}\left(\beta(-4l\xi)-1+\right.\right.$$

$$\left.\left.\left.+\frac{1}{l\xi}\left(0{,}5\beta_1^+(-4l\xi)-\beta(-2l\xi)\right)\right)\right]\right\};$$

$$\psi_{23}(\xi)=\frac{3K}{\xi}\left[\frac{1}{l\xi}\beta_1^-(-2l\xi)-\beta_1^+(-2l\xi)\right]\beta_2^-[\alpha(\xi),\ -2(l+d)\xi];$$

$$\psi_{24}(\xi)=\frac{3K}{\xi}\left[\frac{1}{l\xi}\beta_1^-(-2l\xi)-\beta_1^+(-2l\xi)\right]\beta_2^-[\alpha(\xi),\ -2d\xi];$$

$$\psi_{31}(\xi)=(K_\lambda-1)\psi_{31}^{*}(\xi);\ \ \psi_{32}(\xi)=\frac{K_\lambda-1}{\xi^2}\left[\frac{2}{l}-\left(\xi+\frac{1}{l}\right)\beta_1^+(-2l\xi)+\right.$$

$$\left.+\alpha(\xi)\beta[-2(l+d)\xi]\left(\xi\beta_1^+(-2l\xi)-\frac{1}{l}\beta_1^-(-2l\xi)\right)\right];$$

$$\psi_{33}(\xi)=\frac{K_\lambda-1}{\xi}\beta_2^-[\alpha(\xi),\ -2(2l+d)\xi];$$

$$\psi_{34}(\xi)=\frac{K_\lambda-1}{\xi}\beta(-2l\xi)\beta_2^+[\alpha(\xi),\ -2d\xi];\ \ \psi_{41}(\xi)=(K_\lambda-1)\psi_{41}^{*}(\xi);$$

$$\psi_{42}(\xi)=\frac{K_\lambda-1}{\xi^2}\left\{\frac{2}{l}-\left(\frac{1}{l}+\xi\right)\beta_1^+(-2l\xi)+\alpha(\xi)\beta(-2d\xi)\left[\xi\beta_1^+(-2l\xi)-\frac{1}{l}\beta_1^-(-2l\xi)\right]\right\};$$

$$\psi_{43}(\xi)=\frac{K_\lambda-1}{\xi}\beta(-2l\xi)\beta_2^+[\alpha(\xi),\ -2d\xi];\ \ \psi_{44}(\xi)=\frac{K_\lambda-1}{\xi}\beta_2^+[\alpha(\xi),\ -2d\xi];$$

$$\psi_{11}^{*}(\xi)=\frac{1}{\xi}\{\beta_1^-(-2l\xi)-2l\xi+0{,}5\alpha(\xi)\beta(-2d\xi)[(2-\beta(-2l\xi))\beta(-2l\xi)-1]\};$$

$$\psi_{21}^{*}(\xi)=\frac{\alpha(\xi)}{\xi}\beta(-2d\xi)\left\{1-\beta(-4l\xi)-\frac{1}{l\xi}[1+\beta(-2l\xi)(\beta(-2l\xi)-2)]\right\};$$

$$\psi_{31}^{*}(\xi)=-\frac{\beta_1^-(-2l\xi)}{\xi}\beta_2^+[\alpha(\xi),\ -2(l+d)\xi];$$

$$\psi_{41}^{*}(\xi)=\frac{\beta_1^-(-2l\xi)}{\xi}\beta_2^-[\alpha(\xi),\ -2d\xi];\ \ K=\frac{R}{4l}(K_\lambda-1);$$

$$\beta(\eta)=\exp\eta;\ \beta_1^{\pm}(\eta)=1\pm\beta(\eta);\ \beta_2^{\pm}[\alpha(\xi),\ \eta]=1\pm\alpha(\xi)\beta(\eta);$$

$\Delta_k$ follows from $\Delta_5$ because of replacement of $A_{nk}$ by $A_{n5}$, k, n = 1, 2, 3, 4;

$$A_{k5}=\int_0^\infty J_1^2(R\xi)\psi_{k5}(\xi)\,d\xi;\ \ \psi_{15}(\xi)=\frac{1}{4l\xi}\psi_{11}^{*}(\xi);$$

$$\psi_{25}(\xi)=\frac{3}{4l\xi}\psi_{21}^{*}(\xi);\ \ \psi_{i5}(\xi)=\frac{1}{R\xi}\psi_{i1}^{*}(\xi),\ i=3,\ 4.$$

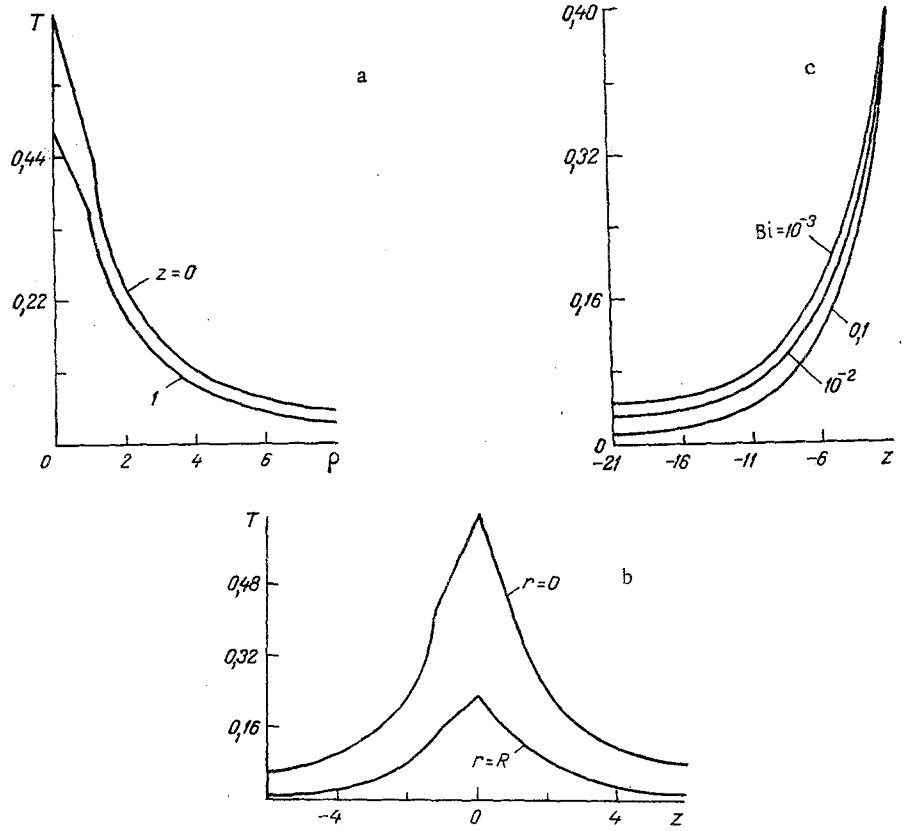


Fig. 1. Dependence of the dimensionless excess temperature T on the relative radial coordinate ρ for Bi = 0.1 (a), axial coordinate Z for Bi = 0.1 (b) and for ρ = 0 (c).

Using the electronic computer ES-1035 computations were performed of the dimensionless excess temperature T for the relative axial $\rho = r/R$ and axial $Z = z/R$ coordinates for the following initial data: $l/R = 1$; $l/d = 1/20$; $K_\lambda = 419$. The numerical results are presented in the figure.

Dependence of the values of T on ρ are shown in the Fig. a. The temperature in the domain of the inclusion $0 \leqslant \rho \leqslant 1$ decreases according to a linear law, where the lines are not parallel for different values of Z. The maximum temperature is achieved at the center of the inclusion ρ = 0, Z = 0 and exceeds the temperature at ρ = 0, Z = 1 by 1.4 times.

Dependences of the values of T on Z are represented in Fig. b for different values of ρ. A symmetric temperature distribution is observed in the interval $-4 \leqslant Z \leqslant 4$. Consequently, the function is $\varphi_j(z, \xi) = 0$ ($j = 2, 3, 4, 5$) and, therefore, (11) is simplified.

The dependence of T on Z is illustrated in Fig. c for different values of the Biot criterion. It is seen that as the heat elimination grows the temperature diminishes for Z < -1 while it is practically independent of the magnitude of the heat elimination coefficient for Z = -1.

## NOTATION

t. temperature field dependent on the cylindrical coordinates r and z; $\lambda(r, z)$, heat-conduction coefficient of an inhomogeneous body; $\lambda_1$, $\lambda_0$, heat-conduction coefficients of the main material and the inclusion; $\alpha_z$, heat elimination coefficient from the surface $z = -l - d$; $K_\lambda = \lambda_0/\lambda_1$, criterion characterizing the relative heat conduction of the body; $s_\pm(x)$, asymmetric unit function; $\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$, Laplace operator $J_\nu(\eta)$, Bessel function of the first kind of order ν.

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# METHOD OF QUASI-GREEN'S FUNCTIONS FOR A NONSTATIONARY NONLINEAR PROBLEM OF THERMAL RADIATION

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We derive a system of two nonlinear integral equations for the determination of a temperature field and the intensity of the incident radiation. The kernels of these equations are expressed in terms of a quasi-Green's function.

One of the methods for increasing the accuracy of thermal calculations consists in converting a boundary value problem of heat conduction to an equivalent integral equation [1]. Various methods can be used for this purpose (see, for example, [2, 4]). In what follows, this conversion is effected with the aid of the method of quasi-Green's functions [5]. The main advantages of this method are: the explicit form of the kernels of the integrand expressions; the incorporation of information relating to the geometry of the domain of integration directly into the kernels using the apparatus of the theory of R-functions [6]. With an appropriate choice of structure for the normalized equation of the domain of integration [6], we obtain Fredholm integral equations of the second kind.

We consider a nonlinear initial-boundary problem for a heat radiating body in which the thermophysical characteristics and heat sources are temperature-independent and in which heat exchange with an external medium is present on a convex surface S (see [7]):

$$\operatorname{div}(\lambda \operatorname{grad} u) - c\rho u_t = -W,\ P \in D,\ t > 0, \tag{1}$$

$$u(P,\ 0) = \psi(P),\ P \in D, \tag{2}$$

$$\lambda \frac{\partial u}{\partial n} + \alpha u = \varphi(P,\ t,\ u),\ P \in S,\ t > 0. \tag{3}$$

Here $\lambda = \Phi(\varphi, t)$ is the thermal conductivity coefficient; c is the specific heat coefficient; $\rho$ is the density of the medium; W is the volumetric heat source or heat sink density,

$$\varphi(P,\ t,\ u) = -\varphi_0(P,\ t) + \varphi_1(P,\ t,\ u),$$

where $\varphi_0(P, t) = q_{source}(P, t) + \alpha u_m(P, t) + \varepsilon\sigma u_m^4(P, t)$ is the total heat flow supplied to S; $\varphi_1(P, t, u) = \varepsilon\sigma u^4$ is the flow radiated in accordance with the Stefan–Boltzmann law. Here $u_m$, in turn, is the temperature of the external medium; $\sigma$ is the Stefan–Boltzmann constant; $\varepsilon = \varepsilon(u)$ is the degree of blackness of surface S.

If surface S contains a concave portion $S_1$ or if there is an exchange of radiative flows with other surfaces, then in the boundary conditions (3) an additional term $\varepsilon \times E$ appears in the function $\varphi_1(P, t, u)$ which accounts for radiation of heat on the concave surface $S_1$, and we then use the integral equations of radiant heat exchange

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